Transfer, a Base for Interaction and Reflective Thinking

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Abstract

In this article the connection between transfer, interaction and reflective thinking is analyzed, especially within the context of learning mathematics. Attention is paid to the question of how to use or to explore the knowledge that has already been acquired. It may be relevant to make a distinction between the "classical" concept of transfer and the concept in which transfer is viewed as constructive and productive, hence not just applied, but elaborated and explored also. Learning mathematics is considered as a cyclic and continuous process and that means that the knowledge that individual students transfer (use and explore) is a motive to negotiate with each other about the merits of the diverse procedures and approaches that the participants were opting for. This dialogue provides an interesting base for critical reflection on the thinking trajectory, approach and insights that each individual student originally preferred and elaborated. Dialogue converts into reflection and therefore reflection is viewed as internalized dialogue. And reflection leads to transfer on a higher level and so on: learning mathematics is a cyclic and continuous process.

Keywords: productive transfer; simultaneous interaction; internalized dialogue; reflection; 'learning mathematics; Gestalt; meaning

Introduction

It is inevitable that, when you have to learn something new or when you have to solve a problem, you make use of the knowledge you already acquired. This process may pass off smoothly without much effort and awareness. In such cases we are dealing with automatized activities, especially when information is used that can be easily re-

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trieved from a well-organized memory. And this is exactly what is going on in everybody's daily functioning. However, it may happen as well that a person has to actively retrieve relevant information. Compare for instance the way professionals generally elaborate their expert knowledge. In these cases, the search for relevant insights is a conscious and sometimes complicated enterprise, because the insights are not just used consumptively, but productively. That implicates that the available knowledge and insights are revisited and elaborated in order to construct new insights, procedures and concepts on a higher level. This process of elaboration is called "transfer" or "transformation", and that means that already acquired knowledge is reconstructed.

However, everybody is acting in his or her own personal way and for this reason it is instructive – in education – to evoke discussion about their individual activities amongst students. This interaction can be called horizontal interaction, while the interaction with the teacher is considered as vertical interaction. Interaction, and so negotiation about everybody's thinking and problem solving processes, may induce interest in one's own thinking. After all, the confrontation with other ways of thinking triggers the question whether other students did perhaps identify more adequate or appropriate thinking trajectories. "What is the difference between the thinking procedures they and I preferred to follow?" This process is called "reflection" and reflection may lead to reanalyzing and improving one's own way of thinking. So-called "poster lessons" could be an excellent device to organize in practice interaction and reflection (we will discuss this later).

Reflection stimulates *transfer* on a higher level, because next time you may ferret out a better and more conscious use of the revisited knowledge that you (interactively) acquired. Moreover, the interaction also exceeds a higher level because in the discussion you use the renewed insights, and these may again be subject of continuing and new reflection. As we see, learning can be characterized as a *cyclic* and *continuous process,* in which transfer, interaction and reflection are closely interwoven. In this article we discuss how these processes are connected, beginning with transfer.

Transfer

When students are using insights and knowledge they already acquired in a positive way during their actual learning, this process tends to be referred to as transfer. According to Hammerton (1987) it can be asked whether learning results in 'benefit of, or impairment from, what has been learned in later performance' (p.781). Hammerton moreover warns: 'Precisely, what it is that transfers, and why it transfers, are far from understood' (p.781).

One salient question is whether there exists a connection between transfer and abstraction. For one can reason: the more abstraction, the more generalization and, hence, the more transfer. If this is the case, learning in contexts may inhibit transfer. A context is, after all, conflated with specific situations and the students may conjecture that what they learn is strictly tied to these situations. Moreover a context appeals to personal experiences related to a specific situation, and not to any other situation. Later on we will discuss the connection between transfer and context by introducing – following Cobb and Bowers (1999) – the concept of 'intercontextuality' and 'situated abstraction'.

Brief historical excursion

But first we entertain a short and selective excursion into the history of thinking about *transfer*. One of the first scholars we meet, is the behaviorist Thorndike (1922). Teachers don't realize, according to Thorndike, how important it is to establish strong connections between stimulus and response. Students should exercise in order to consolidate useful connections and the stronger these connections, the stronger the transfer. For this process Thorndike introduced the term 'incremental learning'.

In *Gestalt psychology* a sudden emerging insight (the "aha-Erlebnis") is considered as an important evocation of transfer. That is conceivable, for this experience is linked up with positive emotions and, hence, easy to identify and to remember. Moreover, when knowledge is acquired on the basis of insight, it is better integrated in the (long term) memory and easier to apply in new (problem) situations. Inert knowledge, on the contrary, is hard to apply to and may even interfere with learning and problem solving. In such cases the former learning triggers negative transfer or may even obliterate what is already learned. An interesting principle in Gestalt psychology is presented as the law of *Prägnanz* ("significance") and that law is formulated as: 'Psychological organization will always be as good as the prevailing conditions allow' (Koffka in Kingma & Vergert, 1993). In every psychological act – reflecting, learning, observing, discussing etc. – Koffka exerts, there is a tendency to make sense, to be complete and to strive for frugality. We don't experience isolated stimuli but a *Gestalt* and that Gestalt is meaningful. So, Gestalt theory claims that the subject is searching for insight, while striving for complete and meaningful structure. This process may trigger fruitful transfer, because the person is elaborating meaningful and applicable knowledge.

In this journal we (Nelissen, 2013) analyzed a comparable process dealing with the emergence of *intuitive thinking*. Intuition, we stated, is characterized as recognition and understanding of structure. In other words, intuition stems from insight in coherence, structure or *Gestalts*. Teachers, for instance, 'know how to act in problematic situations, because they analyze problems on a identified *Gestalt* and that is a meaningful structure', (Nelissen, 2013, p. 36). Gestalts are represented by the subject in essence as (the construction of meaningful) structures or patterns. So, as we can delineate, *intuition* as well as *transfer*, both are tantamount to corresponding processes of construction of Gestalt, while Gestalt is always affiliated with meaning.

Piaget (1977), as we know, never was very optimistic about the impact of (special) training on learning because that would not result in durable, qualitative cognitive change. When the child is capable of reversible and reflective and eventually formal thinking (*"abstraction réfléchissante"*), then we can expect a solid ground for transfer and hence for reorganization of the cognitive structure. This is going on when a child is confronted with the experience that it is not capable of finding the solution for a problem and so has to search for new ways of problem solving. Piaget denotes this process as *accommodation* and what is going on now is that the available cognitive strategies are adapted or transformed and totally revised. In this process of accommodation transfer took place and the afford is a higher level of cognitive structure.

Vygotsky (1978) asserts that education should take place in the context of the so-called 'zone of proximal development'. In daily life the child spontaneously learns 'everyday' words and concepts, while in the intercourse with adults, Vygotsky postulates, 'scientific' concepts are acquired. 'Scientific' concepts are more powerful than 'everyday' concepts because they belong to a well-organized system. That's why they are instruments for transfer. As we see, Vygotsky connects transfer with interaction with adults, especially teachers. Echoing Vygotsky, Davydov (1977) discerns so-called 'empirical concepts' and 'theoretical concepts' and the latter are a rich resource for transfer, because they are general and systematic.

Transfer and context

Research on transfer is usually executed in a laboratory setting where people are requested to perform unnatural and decontextualized tasks not connected with (meaningful) experiences, so Lobato (2006) comments. However, learning acquires sense from experiences and these experiences and intentions of people are ignored in the classical transfer studies. Knowledge is inseparable from the situation in which that knowledge is acquired and elaborated. Knowledge is connected, Lobato states, with culture, social interaction and context and these factors constitute the transfer. But they are neglected in most transfer

studies. In these studies the improvement of the performance of students is what counts. Lobato now suggests an 'actor oriented perspective' indicating that it is relevant to analyze what the previous activities and processes are at work in the learner's mind that regulate the new activities. In the 'situated cognition perspective' the role of context is stressed and Greeno (in Lobato, 2006) outlines that now continuity in activities can emerge; hence, the possibility to go beyond specific, concrete situations (contexts) as a condition for transfer. If meaningful connections are established between contexts and learning, we create 'intercontextuality' and under these conditions transfer is enhanced.

In publications the concept of transfer is sometimes related to the concept of abstraction and the latter concept is considered as the result of a process of 'decontextualization'. No context is supposed to mean: general, broadly applicable, no coincidences, etc. The 'situated cognition' approach, in contrast, advocates that knowledge is always situated: *situativity of* knowledge. No context, according to this approach, implies no situation, and no situation alludes to no meaning, no significance. Nevertheless one should not avoid or even reject the concept of abstraction, and following Cobb and Lobato, we endorse the idea called *situated abstraction*. Symbols and tools are constitutive for meaning and connected to the situation, but at the same time they are connected to mathematical *activity*. This is, Cobb postulates, 'abstraction in context' and this implies the reorganization of already acquired knowledge and insights. As we saw, Piaget denotes such a process of reorganization as accommodation. A third mode of abstraction that Cobb introduces, is called '*collective abstraction*'. Abstraction develops because students cooperate and think as a group according to accepted norms. A community of learners explicitly work out a 'collective abstraction'. Later on we discuss several modes of interaction that stimulates this collective abstraction and transfer.

Transfer and transformation

In the above mentioned 'actor oriented perspective' the researcher is not just addressing statistical, quantitative information, but is interested in the activities of the individual test subjects. The focus is not just on how the subjects transfer knowledge from one task to the other, but on analyzing whether and how knowledge is transformed to activities on a *higher* formal level. This alludes to the work of Russian scholars Vygotsky and Leont'ev, because in their work transfer is considered as a proliferation of social and interactive cooperation. Lobato indicates research (Cobb & Bowers, 1999) that reveals that students reconsider and reconstruct insights when they are confronted with new problems. This mode of transfer is connected with exploration of already learned knowledge and can be referred to as *productive transfer.* Lobato typifies this thinking activity as *transformation.* In this concept of transformation, we recognize Piaget's notion of accommodation because cognitive strategies are transformed or revised. Reproductive transfer, on the contrary, is a relatively simple, however useful, mundane experience that befalls a subject many times a day. After all, people use spontaneously knowledge they had already acquired, although they are generally not aware of this process.

In mathematics education it happens that students – especially low achieving students – scarcely use insights and knowledge they already acquired. They do not realize what they already have learned and know and so they do not actualize and mobilize their knowledge. Hence, transfer, even reproductive, does not function. Let us look at the following example in Figure 1.

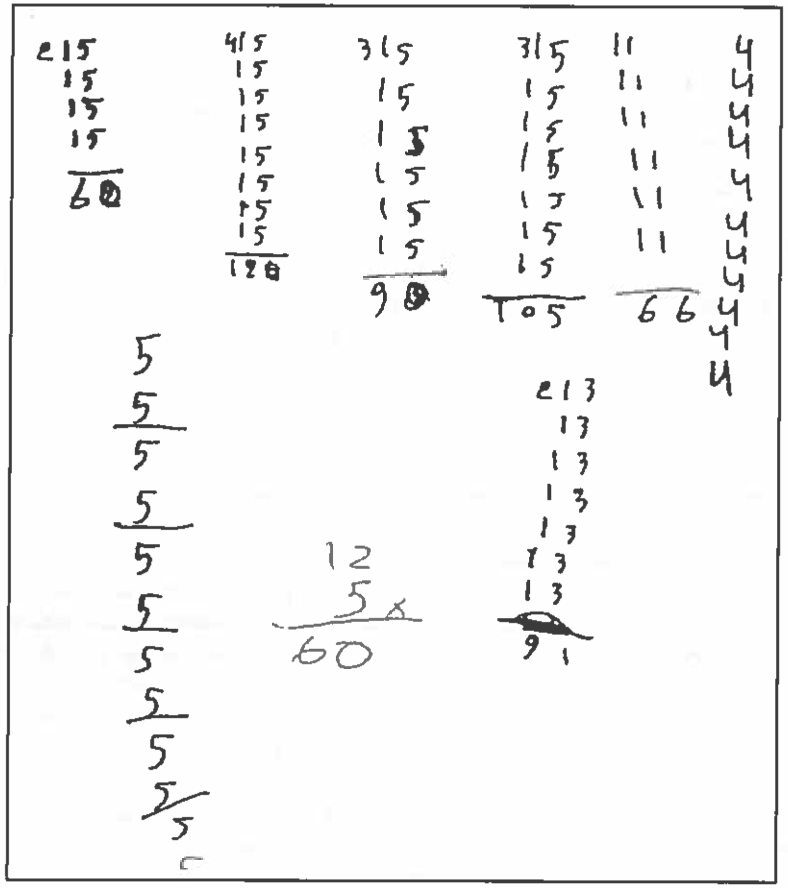

Figure 1

The reader is kindly requested to look for a moment at the work of this thirteen year old student. Is there something that strikes you? Perhaps, yes. The student is performing a multiplication task as: 6 X 15, 7 X 15 and 8 X 15. What we observe, however, is that she does not execute multiplications but additions and moreover, she obviously considers each task as a new and unique task. Students with learning disabilities are often inclined to consider each task or problem as unique. They hardly explore and they are hardly aware of or use what they already did learn and so they do not actualize their knowledge. In short, there is no transfer of knowledge and the more serious the transfer problem, the more the teacher resorts to letting the students learn by *drill and practice*. And, so is the idea of many teachers, the more firmly the information is stored in memory, the easier it can be remembered, transferred and perhaps used.

The social dimension of transfer

As already stated in the introduction of this article, it is instructive to evoke discussion amongst students about the transformation of knowledge (productive transfer) and the various ways they solved a problem. But not only interaction amongst students is fruitful, research suggests that transfer is supported when the *teachers* are also involved in the discussion about problem solving (Engle, in Lobato, 2006). Obviously, transfer is not a process that emerges spontaneously all the time. On the contrary, transfer should be evoked by the teacher's interactive intervention, and Engle denotes this process as *framing participation* or *social framing.* He views interaction in a broad sense and postulates that framing learners as participating in conversation within a larger community helps foster intercontextuality between learning and transfer contexts, because it increases the overlap in participants in both contexts. Greeno (in Lobato, 2006) characterizes this social learning as interactive 'constructive action' and that results in 'connected knowing'. Moreover, Greeno asserts that transfer presupposes *authority*, for a student should have the *courage* to use knowledge that she/he never really explicitly acquired.

But there is more needed to spawn interaction. Granott and Gardner (1994) indicate that at least two aspects influence the interaction. These are the *intensity* of interaction and he *difference* in expertise.

In education there exists of course a range of possibilities when you focus on the intensity of interaction between students and teacher. After all, the more intense the interaction, the more the students are involved. And obviously there exists a difference in expertise (knowledge) between students and teacher. This student-teacher interaction may be characterized as *vertical* interaction. A matter of concern, however, could be that during the discussion students may sometimes be inclined to react to the teacher's requests and evocations in a legitimating sense. That is, students respond in a way they think the teacher is supposed to expect and prefer. It may even happen that they think that the teacher is only controlling whether they were paying attention during the lessons.

Horizontal interaction refers to communication amongst students. When the intensity of the discussion is on a high level, it appears to be an advantage of horizontal interaction (sometimes denoted as cooperative learning) that the students feel free to discuss. However, a disadvantage of the horizontal interaction may be that assertive students overrule the more quiet students. So, when a teacher is organizing learning in groups, it is desirable to compose the groups carefully. That means that the groups should be composed heterogeneously and the norms for cooperating should be discussed and established in advance. Only under such conditions do the weaker students get the opportunity to learn from and cooperate with (high-) gifted students. Interaction on this level is characterized by Mercer (1995) as 'exploratory talk' and that means – unlike the so-called "disputational talk" – that the arguments of all participants are taken seriously and they all contributed to the result of the joint effort.

To complete this enumeration of types of interaction we stress that learning processes, without doubt, take place even in the case that there exists:

a. no interaction, but difference in expertise and
b. no interaction and no difference in expertise.

In the first case there is no interaction and that means that adults do not have any intention to instruct or teach the children. Children – in this case – learn by creative imitation of what they experience in their social daily life, for example their mother tongue, or even a second language, walking, regularity, dancing, quantity, singing, playing games etc. Of course that learning is always supported by the parents or carers.

But even when there is no interaction and even no difference in expertise, children do learn a lot. We are talking now about individual and *independent* learning, more or less apart from adults. But what is the good of this isolated roaming? Well, after all, as Piaget pointed out, (cf. Gopnik, 2009) children are endowed with potential, power and curiosity to perform experiments, to explore their world, and to investigate and to discover.

A third mode of interaction we call *simultaneous* interaction (van Luit, Nelissen & Peltenburg, 2009). In this mode the advantages of horizontal and vertical interaction are reconciled and we will illustrate in the next section how this mode of interaction may be worked out in daily practice.

Simultaneous interaction

Simultaneous interaction can take place in small groups of three or four students. Each group is confronted with the same context problems and in each group the individual participants try to find and to discuss their own approach and possible solutions for the problem. This first phase of the problem solving process can be called the "construction" phase, or perhaps rather the "co-construction" phase, because the students are involved in close cooperation. Of course each individual participant tries to actualize the already acquired knowledge (strategies). The question is what knowledge is applicable or should and could be transformed. This may be the focus of the joint discussion, a social discussion that stimulates, productive transfer. Fosnot and Dolk (2001) qualified this social activity as 'math workshop' – when learners are inquiring, investigating and constructing. In each group the learners register their findings, arguments and ideas on a poster and in front of the classroom the posters of all the groups are exhibited. That's why we typify these lessons as *"poster lessons"*. After the small group discussions have come to an end, the groups get the opportunity to study all that is reported on the posters. Now each sub-group illustrates and explains to the whole class the approach their group advocated and followed by a discussion with the class, supported and supervised by the teacher.

We once organized such a "poster lesson" and the central problem we introduced was: in what bottle – on a table several bottles of diverse shape and size were presented – do you think you can pour the most liquid? After this introduction we explained the working procedure. We requested the learners, who were about eight years old, to look at and think about the problem and to discuss the approaches that were favored (van Luit, Nelissen & Peltenburg, 2009). We asked them to try to work out a shared approach, to write down all ideas that emerged and to discuss these ideas in each group. We told them that a poster (like a scratch paper) can be helpful as a resource for investigation and discussion. We explained what our role, i.e. that of the teacher, would be while the students were collaborating: not intervening, but standing by to support in case there is a problem or a need for information. After that explanation each group started to try to solve the problem. After some time the sub-groups were asked to present their work to the whole class. The solutions for the bottle problem in the groups were of different originality and quality. For instance, in one group it was suggested to weigh the bottles, empty as well as full of water. Another group preferred to pour the full bottles out on the ground and observe where the biggest puddle of water was. In other groups the idea was suggested to use a small glass and to measure how many glasses one can pour into each bottle. During the whole class discussion about these approaches, it appears always to be useful to pay special attention to quiet students; questions by the teacher may be a stimulus for these students to negotiate with the other students about their ideas.

In the whole class discussion, guided by the teacher, the groups informed each other about their findings. The discussion is a resource of inspiration for the thinking activities and so a resource to enhance the problem solving skills and the interactive skills. Moreover, reflection is stimulated when the teacher looks back together with all students on the merits of the different proposals. The proposals are discussed and the learners are requested to review critically their *own* strategies that they originally opted for. This critical analysis of one's own thinking is called *reflection* or reflective thinking.

To summarize, problem solving is affiliated with transfer (or transformation). This process is started by using or reconstructing already acquired knowledge. In the next phase students may discuss this act of transfer (or transformation) with other learners or with the teacher. The question is now whether other and perhaps more adequate thinking strategies arise. This second phase we call *interaction* while the third phase is called the *reflection* phase. Reflection stimulates transfer on a higher level, because reflectively revisited insights are elaborated in the future on this higher level. In the next section we focus on this third phase of reflection.

Reflection

In this article reflection is considered as the human capacity or function to plan, to analyze, to revise, review and change (improve) one's own thinking activities. When reflecting, students learn to critically analyze their own actions. So, reflection may be typified as analyses of one's *own*, personal thinking, rather than the thinking of somebody else or thinking about a problem or task (object). The Russian scholar Zak (Zak, pers. communication, March 7, 1979) presented this idea in a simple diagram (figure 2). Note that Zak uses the term 'theoretical' thinking, echoing Davydov (in Nelissen and Tomic, 1996), indicating, roughly, thinking and reasoning as is typical for the science in question (in this case: mathematics).

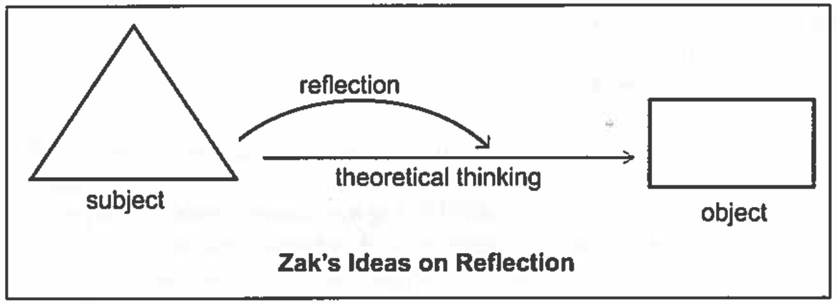

Figure 2

Reflection, we believe, is not a general but a domain specific activity, an activity that is tied to the concepts, procedures and the characteristics of a specific science. This implies that reflection can only be performed when a subject is familiar with the relevant core concepts and thinking strategies of a certain discipline; that is what Davydov typified as theoretical thinking. After all, the correct elaboration of these concepts and strategies is under analysis and, if necessary, revision. Research revealed (Lesh, 1985) that general strategies play a subordinate role in solving math problems, whereas *specific* procedures in mathematical thinking are indispensable for reflection and problem solving. It may be stressed that by reflection the students become less dependent on their teacher. Moreover, reflection builds self-confidence by allowing pupils to discover what they think and why.

It may be useful to distinguish *three modes* of reflection.

At first there is the ability to motivate and analyze a plan of activities that may be executed to try to tackle the problem. The subject evaluates several relevant strategies and this process we call *preliminary* reflection. The core question is how to approach the problem and this process is identified by Sacharova (1982) as a form of prognostic monitoring.

The second mode of reflection occurs when a student is analyzing the thinking activity *during* the process of problem solving to assess whether this process will be relevant and successful. Sacharova characterizes this as monitoring during the solution process.

The third mode of reflection is focused on the final result of the problem solving process. The student now critically reviews the result of this process *afterwards*. The core question is whether this is the result one was striving for and Sacharova identifies this as monitoring of the result of the action.

Research questions

When students learn mathematics and learn how to solve math problems they use and explore the knowledge they already acquired. It is worthwhile and interesting to negotiate with other students about their explorations, because this may trigger arguments to compare all the emerging thinking strategies, including of course their own. The process of analyzing one's own thinking activities we call *reflection*, and we want to stress that negotiation and interaction are indispensable for the emergence and growth of reflection. Before we discuss our own research on this theme, i.e. the connection between interaction and reflection, (Nelissen, 1987), we present a brief and selective summary of the literature on this special subject (for a more comprehensive discussion of the American and Russian research literature we refer to Nelissen & Tomic, 1998)

Let us begin with the question whether reflection is a *general* or a *specific* skill. American psychologists are divided on this question. Some (Brown, 1980) point out the general nature of reflection (metacognition) and others (Siegel & Richards, 1982) see reflection (metacognition) more as a domain-specific skill. Russian psychologists emphasize reflection as general action linked with problem-solving (Semenov & Stepanov, 1979) or as linked with theoretical thinking, general and formal action (Zak, 1984).

Another question then is whether reflection is related to *general* or to *specific* knowledge. Russian psychologists point out its connection with general, theoretical knowledge (Zak, 1984) while others investigate the link with specialist knowledge. Doblaev (1984) and Ajdarova et al. (1983), for instance, investigated reflective thinking in relation with language development and language education. American psychologists underline the relationship to knowledge that is not domain-specific (Gagné, 1983).

Is reflection performed as a *unconscious* process or as a *conscious* activity? Most Russian scholars emphasize the consciousness of reflection. American psychologists are split on this question. Some consider metacognition as a conscious activity (Flavell, 1976) and others see metacognition as the unconscious nature of 'executive processes' (Lawson, 1984).

Can reflection be *developed* by education and training or are we dealing with a stable and *constant* ability? Russian psychologists strongly emphasize the link between education and cognitive development, hence reflection. However, results of education are observable in the long term and not as a result of short-term training (Doblaev, 1984; Ajdarova et al., 1983). American researchers were increasingly optimistic about the possibility of acquiring metacognitive skills through training. But it is also stressed that there is a link with domain-specific structures and that results of education may only be expected in the long run (Gray, 1991).

Reflection: cognitive and non-cognitive

Is reflection purely a *cognitive* process or may be *non-cognitive* aspects involved as well? Russian psychologists (in Nelissen & Tomic, 1996) emphasize the connection between reflection and non-cognitive personality variables. Semenov and Stepanov (1979) make a distinction between intellectual and personal reflection. Intellectual reflection involves concentration on the objective data of a problem, i.e. the problem as such. People ask themselves, for instance, with what type of problem they are dealing and how this can be tackled. A person engaged in *non*-cognitive, personal reflection attributes *meaning* to his/her own actions. Semenov et al., demonstrated in experiments that this personal component of reflection can be encouraged by suggestions as "Solve the problem while thinking out loud. If you think the solution is correct, try to put the solution principle into words". The experiments showed that the participants (students) now became more aware of the difficulties in solving a problem. The researchers could identify three *forms* of personal reflection. Situational reflection permits the participant to become more aware of his or her own meaningful actions. Retrospective reflection means that the participant will begin to review his or her own actions. The third form is called perspective reflection and now, in this evident conflict situation, the participant will generate new plans of action.

The Russian researchers also identified three *forms* of intellectual reflection. A person engaged in extensive reflection only reports on his/her own actions, rather than analyzing them and, if necessary, improving them. Those engaged in intensive reflection (the second form) and above all constructive reflection (the third form) actively intervened in and transformed their own actions. Finally, this classification has led Semenov et al., to distinguish between *productive* and *reproductive reflection*. Productive reflection is characterized by a high (constructive) level of intellectual and a high (perspective) level of personal reflection. Personal reflection plays a dominant role because searching for perspectives forces the person to concentrate on future actions. 'Without personal (perspective) reflection, intellectual (constructive) reflection becomes difficult to achieve', Nelissen & Tomic (1996, p. 47).

Stepanov and Semenov (1985) recommend training children to attribute *meaning* to their actions. If necessary, any stereotypes that occur can now be discredited by challenging them in a conflict situation. Sacharova (1982) affirms this recommendation and moreover centers on the connection between reflection (especially monitoring) and processes of attribution (self-evaluation). This process emerges when someone assesses the difficulty of the task related to their own ability to perform it.

In experiments performed by Bozmanova and Sacharova (1982) it was demonstrated that the higher the level of reflection (self-evaluation), the more careful students were in assessing their own ability in relation to the difficulty of the problem. They were somewhat reluctant and only rarely underestimated the difficulty of the problem they were to solve. However, the *lower* the level of reflection, the more certain these students were of their own abilities and the more they were inclined to underestimate the difficulty of the problem. Students who did *not* reflect at all were unable to solve the problems they previously judged to be easy.

So, research shows that it is obvious that reflection is composed of cognitive as well as non-cognitive aspects.

Reflection as internalized dialogue

It was the Russian psychologist Matis (1982), who studied the relationship between *interaction* (discussion) and reflection. The students who were involved in the experiment were given various tasks to work on and were told to come up with a common result. However, that would only be possible if they consulted one another, but they were also instructed to continue individually with what another student had already started. The task consisted of writing texts and checking each other's working methods and texts. A research result was that not only had the reciprocal monitoring increased, but – perhaps more strikingly – that *self*-monitoring, and therefore reflection, also improved. So the students had learned to evaluate their own actions from the perspective of *another.* This reflection consisted of inserting corrections in their own texts and the students spontaneously began to compare their own approach to those of the other students. Matis concludes that interaction allows students to develop the ability of 'evaluating their own work from the perspective of another' (Matis 1982, p. 283). Matis' conclusion is similar to the research results achieved by Polivanova (1978) who considers interaction as an important stimulus for self-monitoring, or reflection. All these conclusions reinforce our conception that reflection can be considered as *internalized dialogue.*

Such a conception – that reflects in essence a Vygotskian point of view – was endorsed in our own research (Nelissen, 1987) as well. Two

groups of students (about twelve years old) were compared. The experimental group had been taught mathematics according to the "realistic method", meaning that the instruction was characterized by simultaneous interaction and problem-solving within a rich context. Through interaction the students were encouraged by the teachers to reflect on their own thinking processes and approaches. The students in the control group of our research were more likely to concentrate on routine and standard algorithmic approaches and solutions, which was typical for the school curriculum they followed. The students in the experimental group had learned and experienced to comment regularly on each other's actions. As a result of this negotiation they spontaneously began to *anticipate* on the comments of the other students and the teacher. Hence, they became familiar with one another's comments and interiorized these comments, that means transited and transformed the comments in self-evaluation. This we conceived of as reflection, and reflection we typified as *internalized* dialogue. The problem-solving processes of the students in the control group, however, had been based primarily on prescribed standard algorithms without much negotiation, and so there existed no dialogue that could be internalized. For these students there emerged scarcely any reason to reflect on their thinking processes. They had just learned to follow the prescribed rules and were used to and satisfied with this.

A third research project that illustrates the connection between interaction and reflection is a large-scale educational experiment on the mother tongue (Russian language). Ajdarova (et al., 1983) investigated how young students acquire understanding of their own learning activity, i.e. whether they learned to reflect. Starting the experiment, the students discovered that one can study one's own language. They learned that there is always someone who says something (the sender) and someone who listens (the receiver). And of course, there is a message. This message has both form and content. For all these dimensions the students were requested to construct their own symbols (for an example, see figure 3).

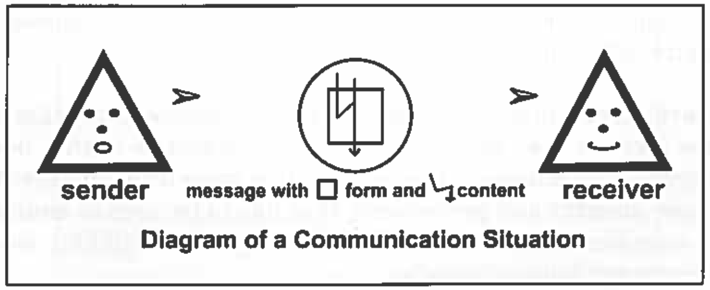

Diagram of a Communication Situation

Figure 3

The students were also confronted with the following problem. Imagine that we live in prehistoric times and language had not yet been invented. How would you let me know that, for instance, fire has broken out and we are in danger? The students discovered that you can manage it in various ways. They discovered that the message may have one meaning but can be interpreted in various ways (sense). They were asked to construct diagrams representing situations and to discuss which diagram gives the best view of the situation. Reflection was elicited through *dramatization* in the following way. The students designed masks for various characters in the play. They created symbols which stood for the meaning (content) and the form of the message (figure 3). Some students were actors, while others designed the scenario and a third group was the audience. The latter group was to decide whether the meanings were clear, so whether the play went according to the scenario. The roles changed each time. The students who were actors in the first round, wrote the scenario in the second, and so on. By changing roles, the students learned how to look at the play each time through a different person's point of view. Moreover, and maybe more importantly, they learned to look at their *own* thinking activities through *another person's* eyes. The construction of diagrams, based on "theoretical thinking" in the terminology of Davydov (1982), led to discussion. And this discussion transformed in reflection.

Hence, Ajdarova's research supports the idea that reflection can be viewed as internalized dialogue. This point of view was already confirmed and advocated, as we saw, in the research of Matis, Polivanova and Nelissen. Moreover we can refer to the research project executed by Doblaev (1984), who trained students of several ages to consider and invent critical questions to analyze texts. Doblaev's research suggests that the training allowed the students to learn to reflect on their activities. Moreover, *transfer* occurred when the students had to analyze entirely new types of texts.

Concluding remarks

In this article we discussed the nexus between three psychological core concepts in the context of learning mathematics: transfer, interaction and reflection.

In learning and problem solving a person always explores knowledge that has already been acquired in the past. It can be useful, however, to relinquish and moreover to elaborate this knowledge in order to *construct* new insights and procedures, that has to be used to analyze and solve a complicated task or problem. echoing Lobato (2006), we called this process productive transfer.

Since, however, every individual will act in her/his own personal way, it is interesting and challenging, in education, when students negotiate about these diverse thinking processes. Evoked by the teacher, the question is discussed which of the thinking trajectories that emerged in the group are the most fruitful. The teacher asks the students to analyze and discuss these trajectories. Moreover, they compare these critically with the trajectory each individual student originally devised and wanted to use. This process we identified as internalized dialogue and the result of this process we called: reflective thinking. The reader may recognize the theory of Vygotsky (1978), for it was Vygotsky who stated that all higher, individual functions (speech, thinking, reflection and so on) are originally social functions.

Reflection is an essential feature of mathematical thinking (Nelissen, 1999) 'and can be considered as a foundation for the development of processes of transfer and generalization', (van Luit et al., 2009; 138). Productive transfer is the basis for interaction on a higher level. Higher, because improved and more relevant concepts and mathematical strategies have been constructed. And this interaction leads, again, to reflection while reflection is the basis for future transfer on a higher productive level. After all, reflection stimulates the elaboration and awareness of the procedures and concepts a person constructs, or reinvents and wants to use, and so on.

Hence, we are dealing with a *cyclic and continuous process* and this process, i.e. the ongoing connection between productive transfer, interaction and reflection is represented in Figure 4.

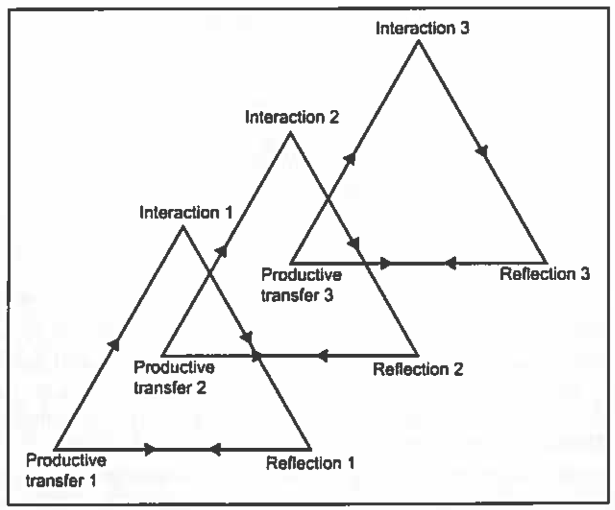

Figure 4

References

Ajdarova, L.I., Gorskaja, Ja., & Cukerman, G.A. (1983). First graders investigate their mother language. *Pedagogische Studiën*, Holland. (Translation from Russian: Z. Nelissen-Bradová).

Bozmanova, M. E. & Sacharova, A. W. (1982). *Besonderheiten der Reflexion als Psychischer Neubildung.* (Reflection as a new mental development in learning). In W.W. Davydov, J. Lompscher & A.K. Markova (Eds.) *Ausbildung der Lerntätigkeit bei Schülern* (Development of learning activities in students). Berlin, Germany: Volk und Wissen.

Brown, A. L. (1980). Metacognitive development and reading. In R.J. Spiro, B. Bruce & W. Brewer (Eds.), *Theoretical Issues in reading comprehension.* Hillsdale, N.J.: Lawrence Erlbaum Ass.

Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher. 28*(2), 4-15. http://dx.doi.org/10.3102/0013189X028002004

Davydov, V. V. (1982). Ausbildung der Lerntätigkeit [Development of Learning Activity]. In: V.V. Davydov, J. Lompscher, & A.K. Markova (Eds), Ausbildung der Lerntätigkeit bei Schülern. Berlin: Volk und Wissen.

Doblaev, L. P. (1984). *Reading and understanding texts.* Moscow, Translation Z. Nelissen-Bradová, Apeldoorn, Holland.

Flavell, J. H. (1976). The metacognitive aspect of problem solving. In L.B. Resnick (Ed.), *The nature of intelligence.* Hillsdale, NJ: Lawrence Erlbaum Ass.

Fosnot, C. T. & Dolk, M. L. A. M. (2001). Young mathematicians at work. Portsmouth, NH: Heinemann.

Gagné, R. M. (1983). Some issues in the psychology of mathematics instruction. *Journal for Research in Mathematics Education. 14*(1), 7-18. http://dx.doi.org/10.2307/748793

Gopnik, A. (2009). *The philosophical baby.* New York: Farrar, Straus and Giroux.

Granott, N. & Gardner, H. (1994). When minds meet: Interactions, coincidence, and development in domains of ability. In R. J. Sternberg & R. K. Wagner (Eds.), Mind in context: Interactionist perspectives on human intelligence (pp. 171-201). Cambridge: Cambridge University Press.

Gray, S. (1991). Ideas in pactice: Metacognition and mathematical problem solving. *Journal of Developmental Education. 14*(3), 24-28.

Hammerton, M. (1987) Transfer of training. Gregory *The Oxford companion to the mind.* Oxford/New York: Oxford University Press.

Kingma, J., & Vergert, ten (1993). Het ontwerp van transferstudies (The design of transfer studies). In W. Tomic, & P. Span (Eds.), *Onderwijspsychologie* (Educational Psychology). Utrecht & Heerlen, Netherlands: Uitg. Lemma BV & Open University.

Lawson, M. J. (1984). Being executive about metacognition. In J.R. Kirby (Ed.), *Cognitive strategies and educational performance.* Academic Press Inc.

Lesh, R. (1985). Conceptual analysis of mathematical ideas and problem solving processes. In L. Streefland (Ed.), *Proceedings of the ninth international conference for the psychology of mathematics education, P.M.E. Noordwijkerhout* (vol. 2, pp. 73-97).

Lobato, J. (2006). Alternative perspectives on the transfer of learning: History, issues and challenges for future research. *Journal of the Learning Sciences. 15*(4), 431-451. http://dx.doi.org/10.1207/s15327809jls1504_1

van Luit, J.E.H., Nelissen, J.M.C., & Peltenburg, M.C. (2009). Learning Mathematics by interaction in young students with special educational needs. In J.B. Mottely & A.R. Randall (Eds.), *Early education.* Nova Science Publishers, Inc. New York.

Matis, T. A. (1982). *Wege zur Erforschung psychischer Neubildungen in der gemeinschaftlichen Lerntätigkeit.* (Ways in investigating mental development in common learning activities). In W.W. Davydov, J. Lompscher & A.K. Markova (Eds.), *Ausbildung der Lerntätigkeit bei Schülern* (Development of learning activities in students). Berlin, Germany, Volk und Wissen.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners.* Multilingual Matters Ltd.

Nelissen, J. M. C. (1987). *Kinderen leren wiskunde.* (*Children learning Mathematics*). De Ruiter, Gorinchem Holland.

Nelissen, J. M. C. & Tomic, W. (1993). Learning and thought processes in realistic Mathematics instruction. *Curriculum and Teaching. 8*(1).

Nelissen, J. M. C. & Tomic, W. (1996). Reflection in Russian educational psychology. *Educational Foundations.* (Winter), 1996.

Nelissen, J. M. C. & Tomic, W. (1998). Reflection in education: Mathematics. In J. Zajda (Ed.), *Learning and Teaching.* James Nicholas Publishers, Albert Park, Australia.

Nelissen, J. M. C. (1999). Thinking skills in realistic mathematics. J.H.M. Hamers, J.E.H. van Luit & B. Csapo (Eds.), *Teaching and learning thinking skills.* Lisse, the Netherlands: Swets and Zeitlinger.

Nelissen, J. M. C. (2013). Intuition and problem solving. *Curriculum and Teaching. 28*(2), 27-44. http://dx.doi.org/10.7459/ct/28.2.03

Piaget, J. (1977) *Récherches sur l'abstraction réfléchissant.* Paris: Presses Universitaires de France.

Polivanova, K.N. (1978) *Issledovanija po problenam vozrastnoy i pedagogicheskoy psychologii* (Research into problems of children in developmental and educational psychology (Translation from Russian: Z.Nelissen-Bradová).

Sacharova, A. W. (1982). *Selbstcontrolle und Selbstbewertung in der Ausbildungder Lerntätigkeit.* (Self-control and self-evaluation in the development of learning activities. W.W. Davydov, J. Lompscher & A.K. Markova (Eds.) *Ausbildung der Lerntätigkeit bei Schülern* (Development of learning activities in students). Berlin, Germany: Volk und Wissen.

Semenov, I.N., Stepanov, S.J. (1979). Productivity of personal thinking component in discursive task solution. *Voprosy Psichologii, 6.* (translation: Z. Nelissen-Bradová).

Stepanov, S. J. & Semenov, I. N. (1985). Psychology of reflection: problems and investigations. *Voprosy Psihologii. 1*, 99-114.

Siegel, R. S. & Richards, A. A. (1982). The development of intelligence. In R.J. Sternberg (Ed.), *Handbook of human intelligence*, Cambridge University Press.

Thorndike, E. L. (1922). *The psychology of arithmetic.* New York: MacMillan

Vygotsky, L. S. (1978). *Mind in society.* Cambridge: London Harvard University Press.

Zak, A. Z. (1984). *Development of theoretical thinking in young students.* Moscow: Pedagogika.